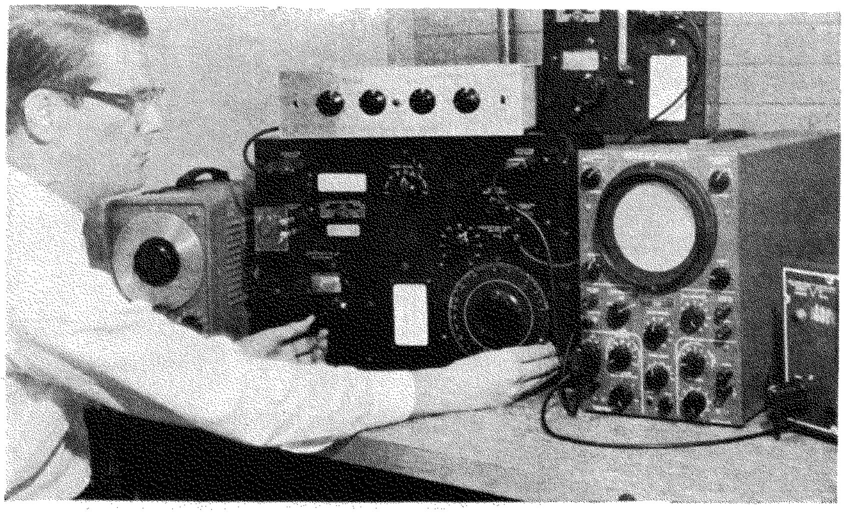

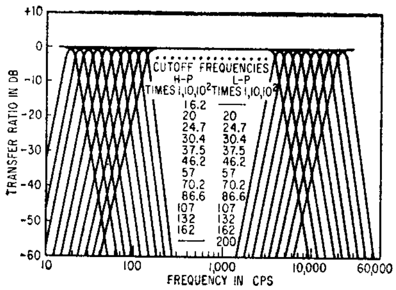

FIG. 1—High-pass and low-pass cutoff frequencies and ideal slopes

Bandpass filter in use in the laboratory, is the light-weight aluminum unit above the signal generator

Active Bandpass Filter

THERE ARE MANY applications for a small, adjustable audio bandpass filter having sharp cut-off characteristics. The filter described here is adaptable for sound analysis[1] and can be designed with fractional octave steps. Sharp cutoff slopes, wide dynamic range and low noise make it valuable for a variety of measurements in the audio field.

By making the filter active, all inductances could be eliminated, reducing size, weight, distortion and hum pickup while extending dynamic range. To obtain sharp corners and high cutoff slopes with a minimum of complication, both high-pass and low-pass sections of the filter were designed to achieve 7th-order Butterworth (maximally-flat) characteristics, giving cutoff slopes of 42 db/octave.

In a 7th-order filter of the type considered, seven elements must be varied simultaneously to alter the cutoff frequency. Because it is difficult to achieve accurate tracking of seven ganged elements, it was decided to change the cutoff frequency by switching the elements in discrete steps. Each decade is divided into eleven intervals, equally spaced on a logarithmic frequency scale. Thus, the ratio between two successive cutoff frequencies is $10^{1/11} = 1.23285$.

The selected frequencies and ideal cutoff characteristics are shown in Fig. 1. There are 34 separate high-pass cutoffs from 16.2 cps to 16,200 cps and 34 low-pass cutoffs running from 20 cps to 20,000 cps. The 12 cutoff frequencies within each frequency decade are obtained by switching resistor values while frequency multiplication factors of 10 and 100 are achieved by switching capacitors.

The absolute value of the input-output voltage transfer ratio, $S(j\omega)$, of a 7th-order low-pass Butterworth filter having unity transfer ratio in the pass region may be written as $|S(j\omega)| = [1 + \omega^{14}]^{-1/2}$, where the cutoff radial frequency ω_0 has been normalized to unity for convenience. As shown elsewhere[2],

Table I—Low-Pass Frequency-Determining Resistor Values

f_0 cps		$d = 1.802$		$d = 1.247$		$d = 0.445$	
	R_0	R_1	R_2	R_1	R_2	R_1	R_2
20	795.82	944.48	446.98	571.73	369.21	596.14	354.15
24.66	645.43	766	362.52	463.69	299.44	483.49	287.22
30.4	523.56	621.37	294.07	376.14	242.9	392.2	232.99
37.48	424.66	503.99	238.52	305.09	197.02	318.11	188.98
46.2	344.51	408.87	193.5	247.5	159.83	258.07	153.31
56.96	279.43	331.63	156.95	200.75	129.64	209.32	124.35
70.23	226.63	268.97	127.29	162.82	105.14	169.77	100.85
86.58	183.83	218.17	103.25	132.07	85.29	137.71	81.81
106.74	149.11	176.97	83.75	107.13	69.18	111.7	66.36
131.59	120.95	143.55	67.94	86.9	56.12	90.6	53.83
162.23	98.11	116.44	55.1	70.48	45.52	73.49	43.66
200	79.58	94.45	44.7	57.17	36.92	59.61	35.42

Table II—High-Pass Frequency-Determining Resistor Values

f_0 cps		$d = 1.802$		$d = 1.247$		$d = 0.445$	
	R_0	R_1	R_2	R_1	R_2	R_1	R_2
16.22	981.1	867.3	1109.8	587.1	1639.3	325.3	976.0
20	795.82	703.51	900.21	476.23	1329.7	263.87	791.68
24.66	645.43	570.57	730.10	386.23	1078.4	214.00	642.08
30.4	523.56	462.84	592.24	313.31	874.81	173.60	520.84
37.48	424.66	375.41	480.37	254.12	709.56	140.80	422.46
46.20	344.51	304.55	389.70	206.16	575.64	114.23	342.72
56.96	279.43	247.02	316.09	167.21	466.90	92.65	277.98
70.23	226.63	200.35	256.36	135.62	378.68	75.14	225.46
86.58	183.83	187.38	207.95	110.01	307.17	60.95	182.88
106.74	149.11	131.82	168.67	89.23	249.15	49.44	148.34
131.59	120.95	106.92	136.82	72.38	202.10	40.10	120.33
162.23	98.11	86.73	110.98	58.71	163.93	32.53	97.60

Use of active elements results in a lightweight, adjustable R-C audio filter having Butterworth attenuation characteristics and 42 db/octave cutoff slopes. Filter supplies more than 50 volts rms output with low distortion and has dynamic range exceeding 100 db. Second-order harmonic distortion is considerably reduced by operating tube heaters at low voltage

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Has Sharp Cutoff

the transfer ratio itself may be written in terms of the complex frequency variable p (equal to $\sigma + j\omega$, where σ is a small constant) as $S(p) = [(p + 1)(p^2 + d_1p + 1)(p^2 + d_2p + 1)(p^2 + d_3p + 1)]^{-1}$, with $d_m = 2 \cos(\pi m/7)$ for $m = 1, 2, 3$. These values of d_m (1.802, 1.247, and 0.445) cause the complex-conjugate poles of $S(p)$ to lie equally spaced on the left half of a unit-radius circle in the p plane with center at $p = 0$. There is also a pole at $p = -1$. This distribution of poles results in Butterworth, or maximally flat, response. For high-pass response, there is a 7th-order zero at $p = 0$ in addition.

One of the easiest ways of realizing the above form of $S(p)$ is to use a separate circuit to achieve each term in parentheses in the expression for $S(p)$. These circuits must, of course, be isolated from each other. The term $(p + 1)^{-1}$ is produced merely by a single R-C time constant. The other terms may be realized in a variety of ways.

Results of previous work[3] lead to the simple feedback circuits of Fig. 2. The active elements with voltage transfer ratios of K should, ideally, have infinite input and zero output impedances. They then give perfect isolation between stages. For practical purposes, cathode followers may be used as long as the required K is less than unity.

Equations relating the d's, K's, and frequency-determining resistor and capacitor values have been given and are discussed in connection with the detailed circuit design of the present filter elsewhere.[2, 3] Tables I and II show the calculated frequency determining resistor values. Resistor values were selected to within 2 or 3 percent of nominal and the capacitor values to within 1 percent, Table III.

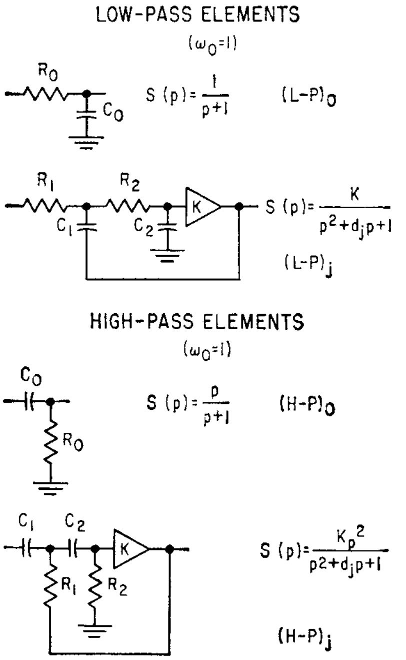

FIG. 2—Elemental frequency-determining circuits with feedback

Table III—Active-Element Transfer Ratios, K, and Frequency-Determining Capacitor (μf) for the Lowest Decade

d	*K*		C_0		C_1		C_2	
	LP	*HP*	*LP*	*HP*	*LP*	*HP*	*LP*	*HP*
.....	0.97	0.97	0.01	0.01	....	....	....	....
1.802	0.97	0.97	....	....	0.015	0.01	0.01	0.01
1.247	0.97	0.97	....	....	0.03	0.01	0.01	0.01
0.445	1.333	1.300	....	....	0.03	0.01	0.01	0.03

Figure 3 shows the circuit of the filter with switches for changing resistor and capacitor values omitted. The switches shown allow the low-pass and the high-pass sections to be used in series, either section separately, or neither section. The low-pass section is entirely direct coupled and could be employed separately as a direct-coupled filter.

Augmented cathode followers

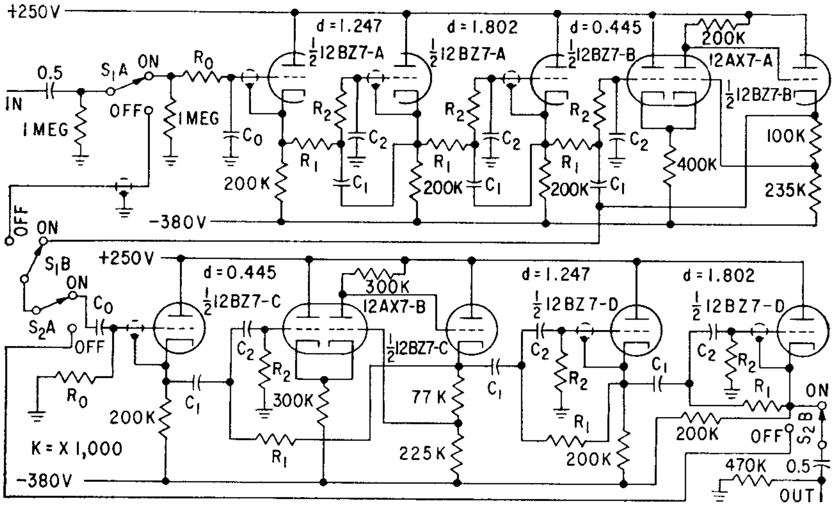

FIG. 3—Filter circuit with switching of frequency-determining elements omitted

(acf)[4] which have low output impedances and voltage transfer ratios which may be made greater than unity are used. The filter was actually designed with all cathode follower *K*'s equal to 0.97 and with the low- and high-pass acf's having *K*'s of 1.33 and 1.3, respectively.

Cathode Followers

The 12BZ7 cathode followers used were found to be superior to 12AT7's in having an input-output voltage transfer ratio nearer unity and a lower output impedance. The factor *K* varied from 0.98 with an added output load of 115,000 ohms to 0.96 with an added load of 15,000 ohms. Because switching of the frequency determining resistors puts a varying load on the cathode-follower circuits, it is desirable to pick the impedance level such that the change from minimum to maximum load alters *K* as little as possible since the *d* factors which determine filter response depend on the *K*'s. On the other hand, with too high an impedance level, the effect of stray capacitances will become important at high frequencies.

The impedance level has been selected so that all cathode-follower *K*'s lie between about 0.982 and 0.97. Frequency-determining resistance values then lie between 1.1 megohm and 32,000 ohms. The actual small variations of *K* with load have been found to exert negligible effect on the filter characteristics. The output impedances of the two acf's used are so low that variable loading has no measurable effect on their *K* values. Values of *K* greater than unity are achieved here by tapping down the feedback line on the output cathode resistor. Distortion is low in the acf circuits and, like cathode followers, they produce no phase inversion.

In the cathode follower circuits, the input shields are driven by the output. Since the output is in phase with and almost equal to the input, this technique reduces the effect of stray capacitance to ground and of capacitance between shield and input appreciably. The minimum input resistance of the filter is about 80,000 ohms. It could be made much greater by using a separate input isolation stage. The output resistance is about 350 ohms. By placing an acf last, it could be reduced to about 5 ohms; it was felt more desirable, however, to use the low acf output resistance to drive frequency-selective elements instead of the output.

Performance

Before measurement of filter performance, the two acf *K*-values were adjusted to give the closest approximation to maximally flat or Butterworth response in the neighborhood of all of the cutoff frequencies. Although the many frequency determining resistance values were selected to within only two or three percent tolerance, it was found that all cutoff regions approximated ideal Butterworth response to within ±1 db and that many were much closer than that to ideal. Only when several of the resistance tolerances were off in the same direction did as much as a 1 db deviation above or below ideal response occur. In the majority of the cases, resistance deviations in opposite directions cancelled out.

The input-output voltage transfer ratio of the low-pass section was found to be 1.33 while that of the high-pass section was 1.26, making the voltage amplification ratio of both sections in series 1.68. These results apply for all positions of the low-pass section but are slightly altered for the $f \times 100$ position of the high-pass section. There, the high-pass voltage transfer ratio is reduced by 4 db compared to the $f \times 1$ and $f \times 10$ positions. This reduction is independent of cutoff position (resistance values) and arises from unavoidable stray capacitance to ground. The effect could have been reduced or eliminated entirely by making all high-pass capacitance values ten times larger and all resistance values ten times smaller. This reduction in impedance level would have caused appreciable change in cathode follower *K's* with cutoff position, however, because of the increased loading and would have necessitated replacement of these cathode followers by acf's if no change of high-pass voltage transfer ratio and corner shape with cut-off position were required.

The impedance level is similarly high in the low-pass section, but voltage amplification reduction in the $f \times 100$ position can be eliminated since the important stray capacitance is there in parallel with the frequency-determining capaci-

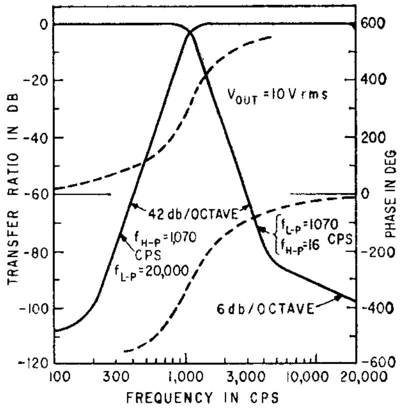

FIG. 4—Typical amplitude and phase characteristics of the filter

tances. By using variable trimmer capacitances in the $f \times 100$ position, the values of all C_0, C_1, and C_2 capacitances can be set once and for all to their correct values, including stray capacitance effects, to yield Butterworth response.

Phase and Amplitude

Typical amplitude and phase characteristics for two different settings of the filter are shown in Fig. 4. Because of unavoidable harmonic distortion in the oscillator used, the high-pass amplitude characteristic with high-pass cutoff frequency $f_{H-P} = 1{,}070$ cps, low-pass cutoff $f_{L-P} = 20{,}000$ cps had to be measured with a wave analyzer. The low-pass characteristic could be measured with either the wave analyzer or a wide-band a-c voltmeter. Phase was measured with a phasemeter. The intrinsic noise output of the filter set for maximum bandpass ($f_{H-P} = 16$ cps, $f_{L-P} = 20{,}000$ cps) was less than 100 μv rms measured with a wide-bandwidth voltmeter.

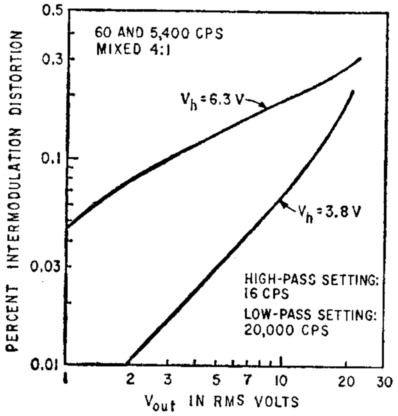

FIG. 5—Intermodulation distortion versus output voltage for two heater voltages

The residual 6 db/octave slope in the low-pass characteristics of Fig. 4 arises from feedthrough. These results suggest that if the impedance level in the low-pass section were reduced by making all frequency determining capacitors larger and all resistors smaller by a factor of ten, an increased region of attenuation of 42 db/octave slope in the $f \times 100$ and $f \times 10$ positions could be achieved.

The amplitude response curves of Fig. 4 are within better than 1 db of ideal Butterworth response over their entire regions until the final high-attenuation curvature sets in. Since the attenuation at cut-off should be 3 db, when both low-pass and high-pass sections are set to the same cutoff frequency the combined attenuation should be 6 db at the peak of the resulting inverted V characteristic. In the large majority of cases, the attenuation is 6 db ±1db; in a few cases, the deviation may be as large as ±2 db.

The upper dashed phase curve of Fig. 4 is associated with the low-pass amplitude curve. It approaches a high-frequency limiting value of 560 deg, while the lower phase curve approaches a value of −560 deg. These phase shifts are appreciable. Others have shown[5] that tremendous phase shifts are required to cause audible effects.

It has already been mentioned that when f_{H-P} and f_{L-P} are set equal, the resulting characteristic is an inverted V with 42 db/octave side slopes. When the high- and low-pass cut-offs are separated by one step, the top of the characteristic is more rounded and is about 1.2 db under the normal transmission of the filter. With two or more steps between high- and low-pass settings, the top of the band-pass characteristic is not reduced compared with the normal transmission and it shows a definite flat portion with three or more steps.

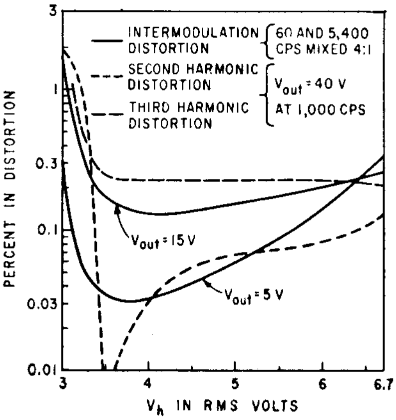

FIG. 6—Intermodulation and harmonic distortion versus heater voltage

The dynamic range of the filter is great. Because of the use of cathode followers and acf circuits, it will handle an output of more than 50 v rms without appreciable distortion. The total dynamic range therefore exceeds 110 db. Having no inductors, the filter is not susceptible to hum pickup from magnetic fields.

Figure 5 shows the measured intermodulation distortion of the filter for two different heater voltages applied to all tubes. The effect of heater voltage in reducing distortion is clarified in Fig. 6. These measurements show that the third harmonic distortion is virtually independent of heater voltage (or current) until such low voltages are reached that cathode emission drops rapidly. Similarly, the fundamental component is independent of heater-voltage until this level is reached. On the other hand, the second harmonic goes virtually to zero just before the point is reached where the emission drops quickly.

Since the second harmonic is the major harmonic component at the lower output voltages. Figs 5 and 6 show that the intermodulation distortion is also appreciably reduced at the lower outputs by reducing the heater voltage to the region where second-harmonic distortion is negligible. This conclusion is also borne out by the different slopes of the two curves of Fig. 5. The low distortion values shown in Figs. 5 and 6 also depend on proper selection of the positive and negative supply voltages.

Second-harmonic cancellation of the above form arises from a dependence of the input-output transfer characteristics of the various tubes of the filter on cathode temperature. At a certain temperature, the curvatures of these characteristics are apparently just right to yield a combined characteristic with no second-order harmonic-generating components over quite a wide dynamic range. Since the specifications of the filter are improved by operation with 3.5 - 3.8 volts on the heaters, it is run at that level.

REFERENCES

(1) H. H. Scott and D. Von Recklinghausen, A Compact, Versatile Filter-Type Sound Analyzer, *Jour Acoust Soc Am* **25**, p 727, July 1953.

(2) J. R. Macdonald, Active, Adjustable Audio Band-Pass Filter, *Jour Acoust Soc Am* **29**, p 1348, Dec. 1957.

(3) R. P. Sallen and E. L. Key, A Practical Method of Designing RC Active Filters, *Trans IRE* **CT-2**, p 74, Mar. 1955.

(4) J. R. Macdonald, Some Augumented Cathode Follower Circuits, *Trans IRE* **AU-5**, p 63, May-June 1957.

(5) J. Moir, Phase Shift and Sound Quality, *Wireless World* **62**, p 165, Apr. 1956.